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A4- Rigid body Mechanics

Oxford.

27/ Pg 158

a) $E = \frac{1}{2} m v^2 = \frac{1}{2} \times 1200 \times 20^2$

$= 2.4 \times 10^5 \ J$

b) $k = \frac{1}{2} I\omega^2 = \frac{1}{2}\left(\frac{1}{2} 25 \times 0.31^2\right)\left(\frac{20}{0.31}\right)^2$

$\left\{ I = \frac{1}{2} mr^2, \ \omega = \frac{v}{r} \right\}$

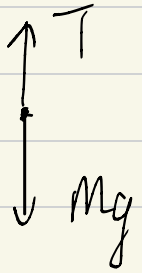
$= 2.5 \times 10^3 \ J$

$\text{Ratio} = \frac{2.5 \times 10^3 \times 4}{4 \times 2.5 \times 10^3 + 2.4 \times 10^5}$

$= 0.04$

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a) Tension in the thread

b) 

$\downarrow F = Ma$

$F = Mg - T$

$Ma = Mg - T$

$a = g - \frac{T}{M}$

$\tau = TR$

$I\alpha = TR$

$I\frac{a}{R} = TR$

$\Rightarrow \frac{Ia}{R^2} = T$

$a = g - \frac{Ia}{mR^2}$

$a + \frac{Ia}{mR^2} = g$

$a\left(1 + \frac{I}{mR^2}\right) = g$

$a = \frac{g}{1 + \frac{I}{mR^2}}$

c) $a = \frac{9.81}{1 + \frac{0.0200}{0.400 \times 0.100^2}} = 1.635 \ ms^{-2}$

d) $v^2 - u^2 = 2as$

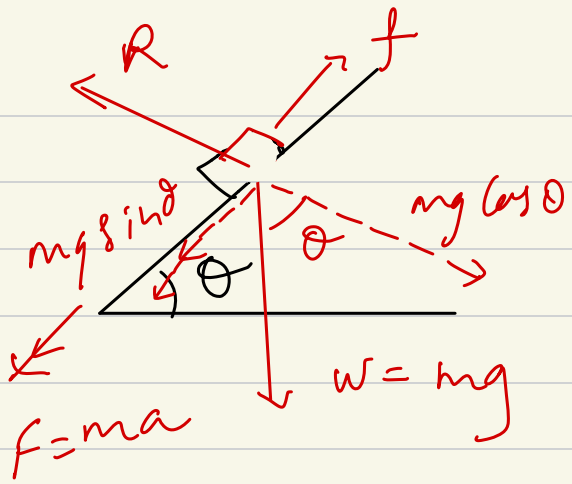
$v^2 - 0 = 2(1.635)(0.600)$

$v = 1.4 \ ms^{-1} \Rightarrow \omega = \frac{1.4}{0.1} = 14 \ rad\,s^{-1}$

$E = \frac{1}{2} I \omega^2$

$= \frac{1}{2} \times 0.02 \times 14^2 = 1.96 \ J.$

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a) $S = ut + \frac{1}{2}at^2$

$1.8 = 0 + \frac{1}{2}(a)(2.9)^2$

$a = 0.43 \ ms^{-2}$

b) Net force = $F = ma$

$ma = mg\sin\theta - f$

friction gives necessary torque due to which ball rotates (rolls)

$\tau = Fd$ & $\tau = I\alpha$ $\Rightarrow F_f = \frac{2}{5}mR^2 \cdot \frac{a}{R \cdot R}$

$F_f = \frac{\tau}{R} = \frac{I\alpha}{R}$

$ma = mg\sin\theta - \frac{2}{5}ma$

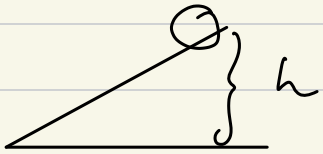
$ma\left(1 + \frac{2}{5}\right) = mg\sin\theta$

$\Rightarrow a = \frac{5g\sin\theta}{7}$

c) $0.43 = \frac{5}{7} \cdot 9.81 \cdot \sin\theta$

$\Rightarrow \theta = 0.06$ rad.

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B Total Kinetic Energy is gained by gravitational potential energy

$\Rightarrow$ TKE = mgh = constant

TKE = Rotational KE + Translational KE.

$$TKE = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2.$$

If TKE is same for both $\frac{1}{2}I\omega^2$ is different as I is different so $\frac{1}{2}mv^2$ is different.

23 Page 154.

a) $\tau = I\alpha = I\frac{\Delta\omega}{\Delta t} \Rightarrow \tau \times \Delta t = I\Delta\omega$

Area under curve $= \tau \cdot \Delta t = 4 \times 1 = 4$

$\Rightarrow 4 = I\omega$

$\Rightarrow \omega = \frac{4}{I} = \frac{4}{0.16} = 25 \text{ rad s}^{-1}$

b) Torque is zero so rate of change of angular velocity $\frac{\Delta\omega}{\Delta t} = 0$, hence $\Delta\omega$ is constant.

c) $\omega = 2\pi f \Rightarrow f = \frac{\omega}{2\pi} = \frac{25}{2\pi}$

No. of revolutions between (1 to 8 s)

$= \frac{25}{2\pi} \times 7 = 27.85 \simeq 28$ revolutions.

d) ΔL from 0 to 1 seconds

$= \tau \times \Delta t = 4 \times 1 = 4$

ΔL between 8 to 10 seconds

$= \frac{-1}{2} \times 4 \times 2 = -4.$

Equal & opposite.

hence it stops.

22 / Page 154. $I = \frac{2}{5} m r^2$

a) $L = I\omega$

$L = \frac{2}{5} 45 \times 10^{-3} \times (2.2 \times 10^{-2})^2 \times 75$

$= 6.53 \times 10^{-4}$ Nms

$\simeq 6.5 \times 10^{-4}$ Nms.

b) $\tau = \frac{\Delta L}{\Delta t}$ Returning ($F = \frac{\Delta p}{\Delta t}$).

$= \frac{6.5 \times 10^{-4}}{15 \times 10^{-3}}$

$= 0.04356$

$\simeq 4.4 \times 10^{-2}$ Nm

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a) $\tau = \frac{\Delta L}{\Delta t} \Rightarrow \Delta L = \tau \times \Delta t$.

(Note :- angular impulse = change in angular momentum).

$\Rightarrow \Delta L = 40 \times 0.1 = 4$ Nms

b) $L = I\omega$

$\omega = \frac{L}{I} = \frac{4}{8 \times 10^{-3}} = 0.5 \times 10^3$ rad sec^{-1}

$\omega = 2\pi f \Rightarrow f = \frac{\omega}{2\pi} = \frac{0.5 \times 10^3}{2\pi}$

$$RPM = f \times 60 = \frac{0.5 \times 10^3}{2\pi} \times 60$$

$$= 4774.648$$

$$\simeq 4800 \text{ RPM (2sf)}.$$

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a) $L_{before} = L_{after}$

$I\omega_{before} = I\omega_{after}$

$I_1\omega_1 + I_2\omega_2 = I_1\omega_1 + I_2\omega_1$

$\Rightarrow \frac{1}{2} \cdot 7 \cdot (0.1)^2 \cdot 80 + 0 = \frac{1}{2} \cdot 7 \cdot \omega_1 + \frac{1}{2} \cdot 3 \cdot (0.25)^2 \cdot \omega_1$

$\Rightarrow 3.5 \times 10^{-2} \times 80 = 3.5 \times 10^{-2} \omega_1 + 9.4 \times 10^{-2} \cdot \omega_1$

$\Rightarrow \frac{2.8}{0.1287} = \omega_1$

$\Rightarrow \omega_1 = 21.76 \text{ rad s}^{-1}$

$\simeq 22 \text{ rad s}^{-1}$ (2sf).

20/b $K_f = \frac{1}{2} I\omega^2$

$= \frac{1}{2}(3.5\times10^{-2} + 9.4\times10^{-2})\,21.76^2$

$= 30.30\ J$

$K_i = \frac{1}{2} I\omega^2 = \frac{1}{2} \times 3.5\times10^{-2} \times 80^2$

$= 112\ J$

$\Delta K = (30 - 112)\ J = -82\ J$

19/ Page 149. Note :- I will change when she puts her arm closure but angular momentum will be conserved

$\Rightarrow \quad L_b = L_a$

$\Rightarrow \quad I_1 \omega_1 = I_2 \omega_2$

$\Rightarrow \quad \frac{I_1}{I_2} = \frac{\omega_2}{\omega_1}$ and $\frac{\omega_2}{\omega_1} = 2.$

$E_1 = \frac{1}{2} I_1 \omega_1^2 \qquad E_2 = \frac{1}{2} I_2 \omega_2^2$

$\frac{E_2}{E_1} = \frac{I_2 \omega_2^2}{I_1 \omega_1^2} = \frac{1}{2} \cdot 4 = 2$

$\Rightarrow \quad E_2 = 2 E_1$

Change in rotational energy $= E_2 - E_1$

$= 2E_1 - E_1 = E_1 \rightarrow$ (B)

as $E_1 = E$. (Question says initial is E).

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a $E_R = \frac{1}{2} I\omega^2$

$= \frac{1}{2} (150 \times 2.5^2)(0.60)^2$

$= 168.75$ J. $\simeq$ 170 J

b $P = \frac{E_R}{t} = \frac{170}{5} \simeq 34$ W.

17 | Page 147

a) $E_R = \frac{1}{2} I\omega^2$

$I = \frac{2}{5} mR^2 = \frac{2}{5} \times 6\times10^{24} \times (6.4\times10^6)^2$

$= 9.83 \times 10^{37}$

for ω :-

radians	Time (second)
2π	$24 \times 60 \times 60$
x	1

$x = 7.27 \times 10^{-5}$

$E = 0.5 \times 9.83 \times 10^{37} \times (7.27 \times 10^{-5})^2$

$= 2.59 \times 10^{29} \simeq 2.6 \times 10^{29}$ J

b Radius of gyration would decrease, so I would decrease, hence E_R would be less.

16/ Page 147 $\omega = 2\pi f$, $f = 1/4$

$\omega = 2\pi \cdot \frac{1}{4} = \pi/2$.

Power = force × velocity = $\tau\omega$

$1.5 \times 10^6 = \tau \times \frac{\pi}{2}$

$\Rightarrow \tau = \frac{1.5 \times 10^6 \times 2}{\pi} = 9.5 \times 10^5$ Nm

15/ Page 147 a $\Rightarrow E_R = \frac{1}{2} I \omega^2$

$I = \frac{2}{3}(0.45)(0.11)^2 = 3.63 \times 10^{-3}$ kgm^2

$E_R = \frac{1}{2} \times 3.63 \times 10^{-3} \times 60^2 = 6.588$ J

$\simeq 6.6$ J.

b $\Rightarrow E_t = 6.6\text{J} = \frac{1}{2} m v^2$

$\Rightarrow v = \sqrt{6.6 \times 2 / 0.45} = 5.416$ m/s

$\simeq 5.4$ m/s

14/ Page 142 a) $\tau = I\alpha$

$I = \frac{\tau}{\alpha}$

$\therefore I = \frac{0.10}{4\pi/3} = 0.0238$

$\Rightarrow I \simeq 0.024$ kgm^2

$\frac{\omega_f - \omega_i}{t} = \alpha$

$\left(\frac{0 - 2\pi \cdot \frac{320}{60}}{8}\right) = \alpha$

$\Rightarrow \alpha = -\frac{4\pi}{3}$

320 RPM

$f = \frac{320}{60}$ RPS

$\omega = 4\pi f$

b) $\omega_f^2 - \omega_i^2 = 2\alpha\theta$

$0 - \left(2\pi \cdot \frac{320}{60}\right)^2 = -2 \cdot \frac{4\pi}{3} \cdot \theta \Rightarrow \theta = .134$

$\therefore$ No. of revolutions $= \frac{134}{2\pi} = 21.326$

$\simeq$ 21 Revolutions

a)

13/Page 142 $\tau = I \cdot \alpha$

for α

$\omega_f^2 - \omega_i^2 = 2\alpha\theta$

radians	Revolution
2π	1
θ	1/4

$\theta = 2\pi \times \frac{1}{4} = \pi/2.$

$\Rightarrow 0 - 23^2 = 2 \cdot \alpha \cdot \pi/2.$

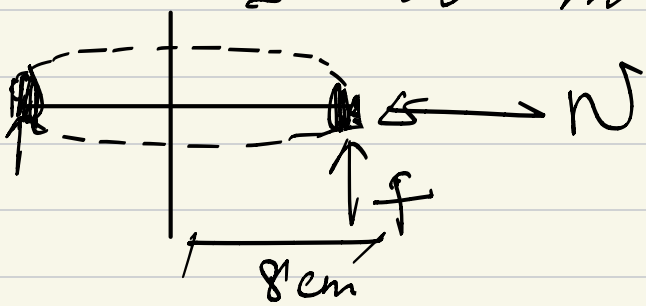
$\Rightarrow \alpha = 168.38 \text{ rad s}^{-2}$

$\Rightarrow \tau = 0.090 \times 168.38$

$\tau = 15.154 \text{ N}$

$\simeq 15 \text{ Nm}$

b)



$\frac{\tau}{2} = fR$

$= (\mu N \times R)$

$\frac{15}{2} = 0.85 \times N \times 0.08$

$N = \frac{15}{0.85 \times 0.08 \times 2} = 110 \text{ N}$

Note :- Torque τ is effected together by two braking pads

$\therefore$ for 1 braking pad

$fR = \frac{\tau}{2}$

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a) $I = \frac{1}{2} mr^2 = \frac{1}{2}(1.2)(0.04)^2$

$= 9.6 \times 10^{-4}$ kgm^2

$\tau = I\alpha$ also $\tau = Tr$

$\Rightarrow I\alpha = Tr$

$\Rightarrow \alpha = \frac{Tr}{I}$ $\Big\}$ $\alpha = \frac{1.8 \times 0.04}{9.6 \times 10^{-4}} = 75$ rads^{-2}

b) $\alpha\pi \rightarrow$ 1 revolution $\Rightarrow$ 5 revolution $= 10\pi$

$\omega_f^2 - \omega_i^2 = 2\alpha s$

$\omega_f^2 - 0 = 2 \cdot (75)(10\pi)$

$\omega_f = 68.64 \approx 69$ rad s^{-1}

c) $s = \omega_i t + \frac{1}{2}\alpha t^2$

$10\pi = 0 + \frac{1}{2}(75)(t)^2$

$t = 0.915$
≈ 0.92 s

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$\tau = FL = I\alpha$

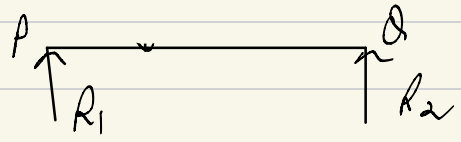
$\alpha = \frac{FL}{I}$

I of system $= m\left(\frac{L}{2}\right)^2 + m\cdot\left(\frac{L}{2}\right)^2$

$= 2 \cdot \frac{mL^2}{4} = \frac{mL^2}{2}$

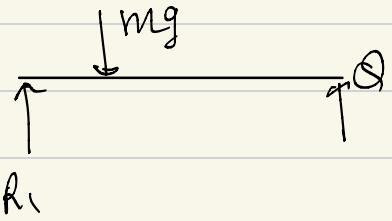
$\Rightarrow \alpha = \frac{F \cdot L \cdot 2}{mL^2} = \frac{2F}{mL}$ $\rightarrow$ (B)

Q 10 / Page 139



before

Moment about Q ⇒ ↺ $R_1 \times 50$.



After

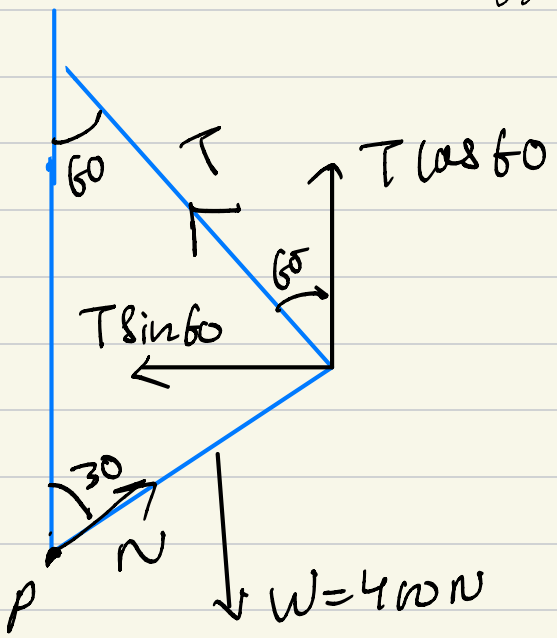
Moment about Q ⇒ ↻ $R_1 \times 50 - mg \times 40$

Net Moment $= 0 \Rightarrow R_1 50 = mg.40$

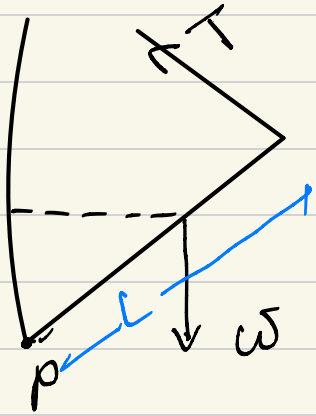
$\Rightarrow R_1 = \frac{mg.40}{50} = 0.8mg$

→ (D)

Q9 / Page 139



Considering moment about P = 0, forces acting :-



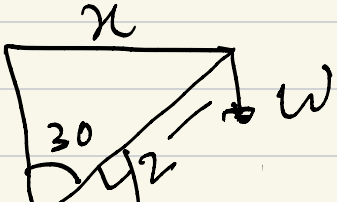
a) let L be the length of rod so W will be at $L/2$ from P.

↻ = W x l distance

@P = W. L sin 30.

↺ = T. l distance

@P



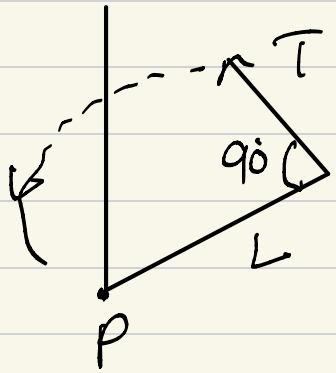
$x = \frac{L \sin 30^\circ}{2}$

$\circlearrowright$ Moment $=$ $\circlearrowleft$ Moment

$\Rightarrow W \frac{L}{2} \sin 30 = T L$

$T = \frac{40 \times 9.81}{2} \sin 30^\circ$

$= 98.1 \simeq 98\,N.$



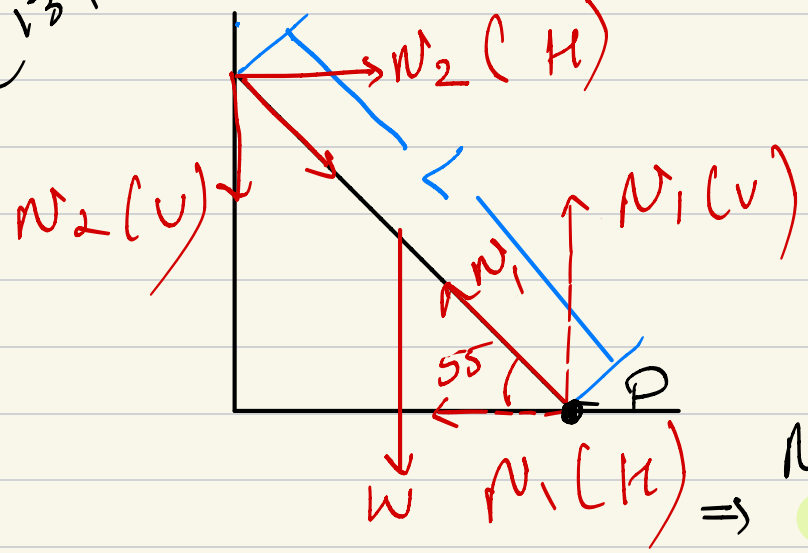
b) $T \cos 60 = 49 \quad (\rightarrow)$

and $W \simeq 40 \times 10 = 400 \quad (\downarrow)$

$\uparrow$ force $\neq \downarrow$ force (due to weight)

(due to tension) hence N must have a vertical component to keep the rod in translational equilibrium.

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a)

$N_2(V) = 0$ as no friction.

so for translational equilibrium,

$N_1(H) = N_2(H)$.

$\Rightarrow T = N_2(H)$

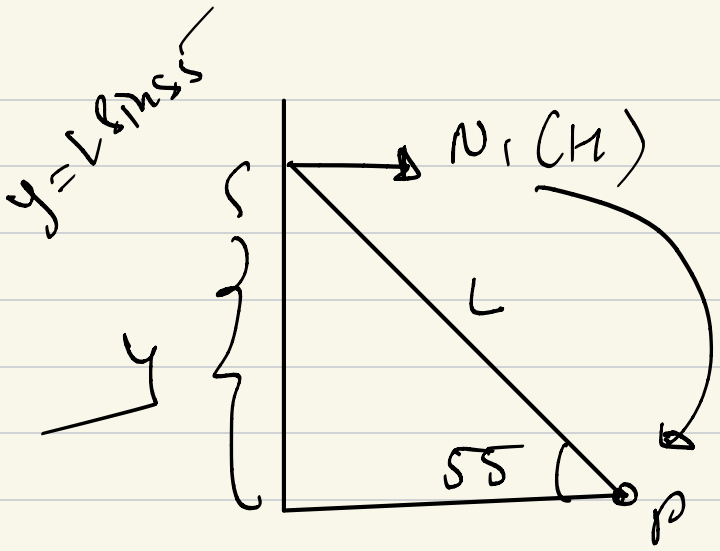
and $N_1(V) = W$.

$\Rightarrow N_1(V) = mg$.

b)

and for Rotational equilibrium.

$\circlearrowright M = \circlearrowleft_{m}$ about P.

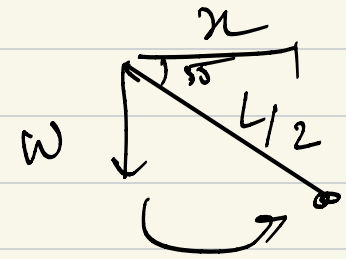


$\circlearrowright_P \quad m = N_1(H) \times y$

$= N_1(H) \cdot L \sin 55$

$\circlearrowleft_P \quad m = w \cdot x$

$= 12 \times 9.81 \times \frac{L}{2} \cos 55$



$x = \frac{L}{2}\cos 55$

$\therefore N_1(H) L \sin 55 = \frac{L}{2} 12 \times 9.81 \times \cos 55$

$\Rightarrow N_1(H) = 41.21$

$\simeq 41\ N$

C highlighted equation

$\Rightarrow f = N_1(H)$

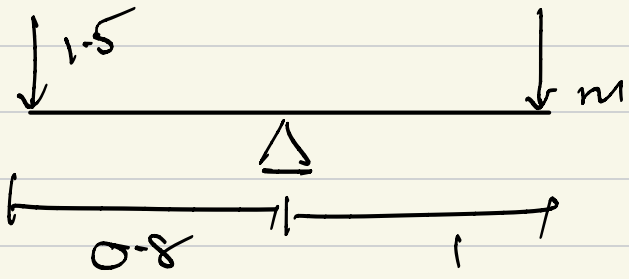
$\Rightarrow \mu N_1(V) = N_1(H)$

$\Rightarrow \mu = \frac{N_1(H)}{N_1(V)} = \frac{41}{12 \times 9.81}$

$\mu = 0.348$

$\simeq 0.35$

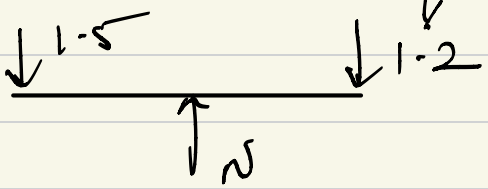
7/ Page 139. a)



Rotational equilibrium @ pivot $\Rightarrow$

$$\circlearrowleft_m = \circlearrowright_m \Rightarrow m(1) = 1.5(0.8)$$
$$\Rightarrow m = 1.2\,kg.$$

b) Translational equilibrium.



$$N = 1.5 \times 9.81 + 1.2 \times 9.81$$
$$= 26.4$$
$$\approx 26\ N$$

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Ball 1 | Ball 2

density is same | density is same

$I = mR^2$ | $I_2 = ?$, $R_2 = R/2$

ρ = same

$$\Rightarrow \frac{M}{V} = \frac{M_2}{V_2} \Rightarrow \frac{M}{M_2} = \frac{V}{V_2}$$

$$\Rightarrow \frac{M}{M_2} = \frac{4/3\,\pi R^3}{4/3\,\pi\left(\frac{R}{2}\right)^3} = 8$$

$\therefore M_2 = M/8$.

$$I_2 = M_2 R^2 = \frac{M}{8}\left(\frac{R}{2}\right)^2 \Rightarrow \frac{MR^2}{32} = \frac{I}{32}$$

$\rightarrow$ (A)

5/Page 137 A → steel | B → lead.

$\rho_B > \rho_A$

a) $m_A = m_B \Rightarrow V_B < V_A$ as $\rho = \frac{m}{V}$

$\Rightarrow R_B < R_A$

$\Rightarrow I_B < I_A \Rightarrow$ ball A has greater I.

{as $I = mR^2$}.

b) $R_A = R_B \Rightarrow V_A = V_B$

$\rho_B > \rho_A$

$\Rightarrow m_B > m_A$ $\left\{\text{as } \rho = \frac{m}{V}\right\}$

$\Rightarrow I_B > I_A$. So ball B has greater I.

4/Page 133 a) $\omega_f^2 - \omega_i^2 = 2\alpha\theta$.

$50^2 - 10^2 = 2(\alpha)40\pi$

$\Rightarrow \alpha = 9.549$

$\simeq 9.5 \text{ rad s}^{-2}$

{for θ,
$2\pi \to 1$ revolution
$x \quad 20$ rev.
$x = 20 \times 2\pi$

b|i $\theta = \left(\frac{\omega_f + \omega_i}{2}\right) \cdot t$

$\Rightarrow 40\pi = \left(\frac{50+10}{2}\right) t$

$\Rightarrow t =$

$\simeq 4.2 \text{ s}$

b|ii $\theta = \omega_i t \pm \frac{1}{2}\alpha t^2$

$2\pi(10) = 10(t) \pm \frac{1}{2}(9.5)t^2$

$\Rightarrow t = 2.73 \text{ s}$

$\simeq 2.7 \text{ s}$

3/ Pg 133 a) Rpm = 1800

RPS = $\frac{1800}{60} = 30$

$\omega = 2\pi f = 2\pi(30) = 60\pi$

$\alpha = \omega / t = \frac{60\pi}{0.1} = 1.88 \times 10^3$

$\Rightarrow \alpha \simeq 1.9 \times 10^3 \ rad\,s^{-2}$

b) $\theta = 5 \times 2\pi = 10\pi$

$\theta = \left(\frac{\omega_f + \omega_i}{2}\right) t$

$10\pi = \left(\frac{0 + 60\pi}{2}\right) t$

$\Rightarrow 20\pi = 60\pi t$

$\Rightarrow t = 1/3 \approx 0.33\ s$

2/ Pg 133 $\left(\frac{\omega_f - \omega_i}{t}\right) = \alpha$

a) $\frac{\omega_f - 0}{20} = 4.7 \Rightarrow \omega_f = 94 \ rad\,s^{-1}$

$\omega = 2\pi f \Rightarrow f = \frac{\omega}{2\pi} = \frac{94}{2\pi}$ RPS

$\therefore$ Rpm $= \frac{94}{2\pi} \times 60 = 896 \simeq 900$ Rpm.

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a) $\frac{\omega_f - \omega_i}{t} = \alpha$

$\frac{15 - 5}{20} = \alpha$

$\alpha = \frac{10}{20} = 0.50 \text{ rad s}^{-2}$

b) $\theta = \omega_i t + \frac{1}{2}\alpha t^2$

$\theta = 5(20) + \frac{1}{2} 0.5 (20)^2$

$\theta = 200 \text{ rad.}$

$= \frac{200}{2\pi}$ Revolutions

$= 31.8$

$\simeq$ 32 revolutions