AA - HL - Vector applications -
selected questions from “ Haese “

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A cable car leaves from $(10,\ 3,\ 0)$. It moves in the direction $2\mathbf{i} - 2\mathbf{j} + \mathbf{k}$ at $4.5\ \mathrm{m\,s^{-1}}$.

a Find the velocity vector of the cable car.

b Find the position of the cable car after 30 seconds.

c The destination of the cable car has X-coordinate 550. Find the length of the cable car ride.

d At what angle to the horizontal does the cable car travel?

a) $\frac{4.5}{\sqrt{9}}\begin{pmatrix}2\\-2\\1\end{pmatrix}$

$= \begin{pmatrix}3\\-3\\1.5\end{pmatrix}$

b) $\bar{v} = a + t(b)$

$= \begin{pmatrix}10\\3\\0\end{pmatrix} + 30\begin{pmatrix}3\\-3\\1.5\end{pmatrix}$

$\bar{v} = \begin{pmatrix}10+90\\3-90\\0+45\end{pmatrix} = \begin{pmatrix}100\\-87\\45\end{pmatrix}$

c) $550 = 10 + t(3)$

$\Rightarrow \frac{540}{3} = t$

$\Rightarrow 180 = t$

$y = 3 - 180(3)$

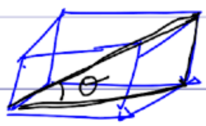
$y = -537$

$z = 0 + 1.5(180)$

$= 270$

d: $\sqrt{540^2 + 540^2 + 270^2} = 810\text{m}$

d)



$\sin\theta = \frac{opp}{hyp} = \frac{270}{810}$ $\theta = 19.5°$

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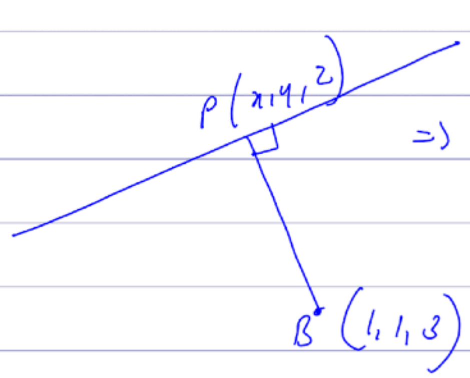
a Find the coordinates of the foot of the perpendicular from $(1, 1, 3)$ to the line with vector equation $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix} + t\begin{pmatrix} 2 \\ 3 \\ 1 \end{pmatrix}$, $t \in \mathbb{R}$.

b Hence find the shortest distance from the point to the line.

a) let co-ordinate of P be (x, y, z).



P ∈ line

$\Rightarrow P\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix} + t\begin{pmatrix} 2 \\ 3 \\ 1 \end{pmatrix}$

direction $\vec{BP} = \begin{pmatrix} 1+2t-1 \\ -1+3t-1 \\ 2+t-3 \end{pmatrix} = \begin{pmatrix} 2t \\ 3t-2 \\ t-1 \end{pmatrix}$

$\vec{BP} \perp l \Rightarrow \vec{BP} \cdot \vec{l} = 0$

$\Rightarrow \begin{pmatrix} 2t \\ 3t-2 \\ t-1 \end{pmatrix} \cdot \begin{pmatrix} 2 \\ 3 \\ 1 \end{pmatrix} = 0$

$\Rightarrow \left.\begin{matrix} 4t + 9t - 6 \\ + t - 1 \end{matrix}\right\} = 0$

$\Rightarrow 14t - 7 = 0$

$\Rightarrow t = 1/2$

$\Rightarrow \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 + 1/2(2) \\ -1 + 1/2(3) \\ 2 + 1/2(1) \end{pmatrix} = \begin{pmatrix} 2 \\ 1/2 \\ 5/2 \end{pmatrix}$

$d = \sqrt{(2-1)^2 + (1/2-1)^2 + (5/2-3)^2}$

$= \sqrt{1^2 + (-1/2)^2 + (-1/2)^2}$

$= \sqrt{6/4} = \sqrt{\frac{3}{2}}$

Discuss the relationship between:

Line 1: $\mathbf{r}_1 = \begin{pmatrix} 3 \\ -1 \\ 2 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ -2 \\ -1 \end{pmatrix}$

Line 2: $x = 1 - 2\mu, \quad y = 4\mu, \quad z = -1 + 2\mu$

Line 3: $x = \dfrac{y-1}{2} = z - 1$

$d_1 = \begin{pmatrix} 1 \\ -2 \\ -1 \end{pmatrix} \qquad d_2 = \begin{pmatrix} -2 \\ 4 \\ 2 \end{pmatrix} \qquad d_3 = \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}$

$\vec{d_2} = -2(\vec{d_1})$ $\therefore$ l_1 and l_2 are $\parallel$ lines.

for l_1 and l_3 $\quad$ l_1 is not $\parallel$ to l_2.

checking for point of intersection:-

let l_1 & l_2 meet @ $P\begin{pmatrix} a \\ b \\ c \end{pmatrix}$.

$\Rightarrow \begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{pmatrix} 1-2u \\ 4u \\ -1+2u \end{pmatrix}$ & $\begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{pmatrix} t \\ 2t+1 \\ t+1 \end{pmatrix}$

$\Rightarrow 1 - 2u = t \qquad \Rightarrow (t + 2u = 1)2 \quad = 2t + 4u = 2$

$4u = 2t + 1 \qquad\qquad 2t - 4u = -1 \qquad\qquad 2t - 4u = -1$

$4t = 1$

$t = 1/4.$

and $u = \dfrac{t-1}{-2} = \dfrac{1/4 - 1}{-2} = 3/2$

Screenshot

checking for 3rd equation:—

$$-1+2q = t+1$$

$$\Rightarrow -1+2\left(3/2\right) = \frac{-2+6}{2} = \frac{4}{2} = 2 \quad \text{and} \quad t+1 = 1/4+1 = 5/2$$

as $2 \neq 5/2$, The lines don't intersect

$\therefore$ l_1 and l_2 are skew lines.

$$\cos\theta = \frac{l_1 \cdot l_2}{|l_1| \cdot |l_2|} = \frac{\begin{pmatrix}-2\\4\\2\end{pmatrix}\begin{pmatrix}1\\2\\1\end{pmatrix}}{\sqrt{2^2+4^2+2^2}\;\sqrt{1+2^2+1}}$$

$$= \frac{-2+8+2}{\sqrt{24}\cdot\sqrt{6}}$$

$$\Rightarrow \theta = \cos^{-1}\left(\frac{8}{\sqrt{24\times 6}}\right) = 48.19 \approx 48.2^\circ$$

Similarly it could be checked for Line l2 and line l3