Section A

1\. a/i $\theta = \dfrac{2\pi}{5} = 1.257$ radians

a/ii $s = r\theta \quad r = \dfrac{s}{\theta} = \dfrac{12}{1.257} = 9.55$ cm

b $A\Delta = \left(\frac{1}{2}\sin\theta \cdot r^2\right)5$

$= \frac{1}{2}\sin(1.257)\cdot(9.55)^2 \times 5$

$= 216.88$ cm².

Δ shaded region $= \pi(9.55)^2 - 216.88$

$= 69.65 \approx 69.6$ cm^2

Q2 $\boxed{1}\ \boxed{2}\ \boxed{3}$

choosing chair person first 10 P,
choosing vice chair person next 9 P,
choosing treasurer last 8 P,
choosing 4 additional members $^7C_4$.
{in any order so C}.

$\Rightarrow 10 \times 9 \times 8 \quad {}^7C_4$

$= 25200$

Q3 a $\bar{x} = 6.45$
{In texas, add x values in statistics tab in doc 1 table then select stats calculations}

b/i $\bar{y} = 0.014375 + 2.1625(6.45)$

$= 13.96$

b/ii $P = 18.48$ using formula and substituting.

$P = 13.96 \times 8 - \{11.8 + 9.7 + \ldots\}$

c using Tinspire. $r = 0.953958$
≈ 0.954

Steps.
in column 1 (name x) enter values.
In column 2 (name as y), enter y values.

then select stats calculations and linear regression mx+c. will give r value.

Q4 $\quad g(x) = a\cos(bx) - 3$

a $\quad a =$ amplitude $= \frac{-8-2}{2} = \frac{-10}{2} = -5$

$b = \frac{2\pi}{\text{Period}} = \frac{2\pi}{32} = \frac{\pi}{16}$

b $\quad g(x+k) = g(x)$

$= -5\cos\left(\frac{\pi}{16}x\right) - 3.$

$k = 1$ Period $= 32.$

c $\quad f(x) = p\sin\frac{\pi}{16}(x-q) \cancel{-3}. = -5\cos\frac{\pi}{16}x \cancel{-3}$

c/i cos x graph gets converted into sine graph with a horizontal translations of $\pm\pi/2$.

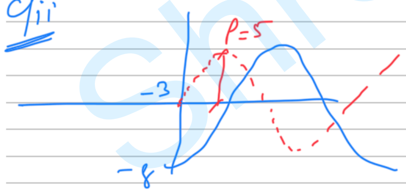
∴ as q is +ve, Period shift.

So $2\pi \quad \pi/2$

$32 \quad ?\ q$

$q = \frac{32 \times \pi}{2 \cdot 2\pi} = \underline{\underline{8}}$

c/ii



as seen in graph $p = 5$.

Q5

17.5	31
22.5	42
27.5	61
32.5	46
37.5	29

using Tinspire lists and spread sheets followed by stats calculations

a/i $\quad \bar{x} = 27.5$

a/ii $\quad \sigma = 6.26374$

≈ 6.264

b $\quad X \sim N(27.5,\ 6.264^2)$

Inverse Normal for $p_1 = 0.75$, $p_2 = 0.25$

Corresponding values $\begin{cases} X_1 = 31.725 \\ X_2 = 23.275 \end{cases}$

$IQR = X_1 - X_2 = 8.45$

Q6 $\quad \sum_{k=0}^{\infty} \left(\frac{53}{1000}\right)\left(\frac{1}{100}\right)^k$

$S_\infty = \frac{a}{1-r} = \frac{53/1000}{1 - \frac{1}{100}} = \frac{53 \times 100}{1000 \times 99} = \frac{53}{990}$

a) $\Rightarrow S_\infty = 5.353535 \times 10^{-2}$

$= 0.0\overline{53}$

b) $\quad 0.2535353 = x$

$\Rightarrow 2.535353 = 10x$

$\Rightarrow 2 + 0.5353 = 10x$

$\Rightarrow 2 + 0.05353 \times 10 = 10x$

$\Rightarrow 2 + \frac{530}{990} = 10x$

$\Rightarrow \frac{2510}{990} = 10x \Rightarrow x = \frac{251}{990}.$

Q7 $A \rightarrow 10\%$, $B \rightarrow 5\%$ } defective

a. $P(X=2)$, $X \sim B(10, 0.10) = 0.193710$ (A)

or

$P(X=2)$, $X \sim B(10, 0.05) = 0.074635$ (B)

$\therefore$ $P(X=2)$ from either machine

$= 0.5 \times (A) + 0.5\,(B)$

$= 0.13417$

b. conditional propability $\left(\dfrac{P \text{ a deffective from } A}{P(X=2)}\right)$

$\dfrac{0.5 \times 0.193710}{0.13417} = 0.72188 \approx 0.722.$

Q8. $f(x) = \sqrt{x^2 - 16}$ $\quad 4 \le x \le 5$

$g(x) = 2 + \dfrac{x^2}{25}$ $\quad 0 \le x \le 5$

$y_1^2 = x^2 - 16$ $\quad (y_2 - 2)25 = x^2$

$y_1^2 + 16 = x^2$

Volume $= \pi\left[\int_0^3 (y^2+16)\,dy - \int_2^3 (y-2)25\,dy\right]$ → y intercept = 2.

$= \pi\left[57 - \dfrac{25}{2}\right]$

$= 139.800$

Q9 $\dfrac{d\theta}{dt} = -\dfrac{\pi}{60}$

$A = 15 \times 20 \times 0.5 \times \sin\theta$

$140 = 150 \sin\theta \Rightarrow \sin\theta = \dfrac{140}{150}.$

Now $c^2 = 15^2 + 20^2 - 2\cdot 15 \cdot 20 \cdot \cos\theta$

$\Rightarrow 2c \cdot \dfrac{dc}{dt} = -600(-\sin\theta)\cdot\dfrac{d\theta}{dt}$

$= -600\left(-\dfrac{14}{15}\right)\cdot\left(-\dfrac{\pi}{60}\right)$

$= -29.3215$

Now

$C = \sqrt{625 - 600\cdot\cos\theta}.$

$\sin\theta = \dfrac{14}{15}$ $\quad \cos\theta = \sqrt{1 - \dfrac{14^2}{15^2}} = 0.3590$

$\therefore C = 20.238.$

$\Rightarrow \dfrac{dC}{dt} = -\dfrac{29.3215}{2 \times 20.238} = -0.7244$

$= -0.7244$ cm/min

SECTION B

Q 10 a. $V_{air} = 60e^{-0.1t}$
$V_{car} = 5t$

when airplane lands $t = 0$

i) $S_{air} = 60$
ii) $S_{car} = 0$

b/i $60e^{-0.1t} = 5t$
$\Rightarrow 60e^{-0.1t} - 5t = 0$
$\Rightarrow t = 6.356$ s

using GDC graph and then where the graph crosses the x-axis.

b/ii $5(6.356) = 31.78$ ms^{-1}

c $d_{car} = \int v\,dt = \int 5t\,dt + c$
$= \frac{5t^2}{2}$

$d_{air} = \int 60e^{-0.1t} = \frac{60e^{-0.1t}}{-0.1} = -600e^{-0.1t} + c'$

$d(t) =$ distance$_{air}$ $- d_{car} + c + c'$
$= -600e^{-0.1t} - 5t^2/2 + c + c'$

at $t = 0$, $d(t) = 100$

$\Rightarrow 100 = 0 - 600 + c + c'$

$\Rightarrow c + c' = 700$

$\therefore d(t) = -\frac{5t^2}{2} - 600e^{-0.1t} + 700$

d using GDC graph, where $d(t) = 0$ or where graph crosses the x-axis.

$\Rightarrow t = 15.06$ seconds

e for car $d(t) = \frac{5t^2}{2}$

$= \frac{5(15.06)^2}{2} = 567$ m

Q11

a) $\int \arccos x \, dx = \int 1 \cos^{-1} x \, dx$

As per ILATE

$\Rightarrow I = \cos^{-1}x \int 1\,dx - \int \left(\frac{d}{dx} \cos^{-1}x \int 1\,dx \right) dx$

$= \cos^{-1}x \; x - \int \frac{-1}{\sqrt{1-x^2}} \, x \, dx$

$= x\cos^{-1}x - \frac{1}{2} \int \frac{-2x}{\sqrt{1-x^2}} \, dx$ $\quad \left\{ \begin{array}{l} \text{let } 1-x^2 = t \\ \Rightarrow -2x\,dx = dt \end{array} \right.$

$= x\cos^{-1}x - \frac{1}{2} \int \frac{1}{\sqrt{t}} \, dt$

$= x\cos^{-1}x - \frac{1}{2} \frac{t^{-1/2+1}}{-1/2+1} + C$

$= x\cos^{-1}x - \frac{1}{2} \frac{t^{1/2}}{1/2} + C$

$= x\cos^{-1}x - \sqrt{1-x^2} + C$

$= x \arccos x - \sqrt{1-x^2} + C.$

b) $f(x) = \begin{cases} 3x \arccos x^2 & 0 \le x \le k \\ 0 & \text{otherwise} \end{cases}$

b) i) $P(x) = \int_0^k 3x \arccos x^2 \, dx = 1$

let $x^2 = t$, $2x\,dx = dt$; $\quad x = 0, t = 0$; $x = k, t = k^2$

$\Rightarrow \frac{3}{2} \int_0^k 2x \arccos x^2 \, dx = 1$

$\Rightarrow \frac{3}{2} \int_0^{k^2} \arccos t \, dt = 1$

$\Rightarrow \frac{3}{2} \left[t \arccos t - \sqrt{1-t^2} \right]_0^{k^2} = 1$

$\Rightarrow \frac{3}{2} \left[\left(k^2 \arccos k^2 - \sqrt{1-k^4} \right) - \left(-\sqrt{1} \right) \right] = 1$

$\Rightarrow \frac{3}{2} \left[k^2 \arccos k^2 - \sqrt{1-k^4} + 1 \right] = 1$

$\Rightarrow k^2 \arccos k^2 - \sqrt{1-k^4} + \frac{3}{2} - 1 = 0$

b) ii) using GDC, $k = 0.71132$.

(Hint: plot graph and find zero).

c) i) $E(X) = x\,P(x)$

$= \int_0^k x \cdot 3x \arccos x^2 \, dx$

$= \int_0^{0.71320} 3x^2 \arccos x^2 \, dx$

$= 0.456$ $\quad$ {using GDC integral}

$Var(X) = \int_0^k 3x^3 \arccos (x^2) \, dx - (E(X))^2$

$= 0.02899 \approx 0.029$

{using GDC}.

d) $P(\mu - \sigma < X < \mu + \sigma)$. $\quad \sigma = \sqrt{0.029}$

$= P(0.286 < X < 0.626$

$= \int_{0.286}^{0.626} P(x) = 0.6189$

≈ 0.619

$P(x) = 3x \arccos x^2$

Q12

$L_1: \; x = 5 + s, \quad y = 4 - s, \quad z = 2 + s$

$L_2: \; x = 2p - 1, \quad y = p + 7, \quad z = 3p - 5$

So
$$5 + s = 2p - 1 \quad \text{or} \quad s - 2p = -6$$
$$4 - s = p + 7 \quad \text{or} \quad p + s = -3$$
$$2 + s = 3p - 5 \quad \text{or} \quad s - 3p = -7$$

Solving 1st two using GDC. $s = -4$, $p = 1$.

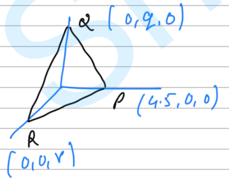
Substitute in third to check: $-4 - 3 = -7$ ✓

$$\therefore \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 8 \\ -2 \end{pmatrix}$$

b|i
$$r = \begin{pmatrix} 1 \\ 8 \\ -2 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix} + \mu \begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix}$$

b|ii $r \cdot \bar{n} = \bar{a} \cdot \bar{n}$.
$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} \cdot \begin{pmatrix} -4 \\ -1 \\ 3 \end{pmatrix} = \begin{pmatrix} 1 \\ 8 \\ -2 \end{pmatrix} \cdot \begin{pmatrix} -4 \\ -1 \\ 3 \end{pmatrix}$$
$$-4x - y + 3z = -4 - 8 - 6$$
$$\Rightarrow 4x + y - 3z = 18$$

c|i



$0 + q - 0 = 18$

$q = 18$

$0 + 0 - 3r = 18$

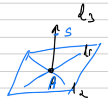
$r = \frac{18}{-3} = -6$

$r = -6.$

d. $\vec{RQ} = \begin{pmatrix} 0 \\ 18 \\ 6 \end{pmatrix}$, $\vec{RP} = \begin{pmatrix} 4.5 \\ 0 \\ 6 \end{pmatrix}$ $\qquad$ Area $= \frac{1}{2} |s_1 \times s_2|$

$\vec{RQ} \times \vec{RP}$
$$\begin{matrix} 0 & 18 & 6 \\ 4.5 & 0 & 6 \end{matrix}$$
$$= \hat{i}(18 \times 6) - \hat{j}(-4.5 \times 6) + \hat{k}(-4.5 \times 18)$$
$$= 108\hat{i} + 27\hat{j} - 81\hat{k}$$
$$|\vec{RQ} \times \vec{RP}| = \sqrt{108^2 + 27^2 + (-81)^2}$$
$$= 137.67$$
$$\text{Area} = \frac{1}{2} \times 137.67 = 68.836$$
$$\approx 68.84$$

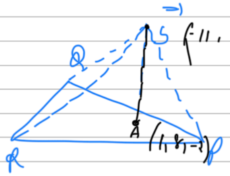
e



$$r_3 = \begin{pmatrix} 1 \\ 8 \\ -2 \end{pmatrix} + v \begin{pmatrix} -4 \\ -1 \\ -3 \end{pmatrix}$$

f $x = 1 + v(-4) \Rightarrow -11 = 1 - 4v$
$\Rightarrow v = 3.$

g pyramid



Height of pyramid $SA =$
$$\sqrt{12^2 + 3^2 + 9^2}$$
$$= 15.297$$

Volume of pyramid $= \frac{1}{3}$(volume of Δ prism)
$$= \frac{1}{3}(\text{c/s area} \times \text{height})$$
$$= \frac{1}{3}(68.84 \times 15.297)$$
$$= 351.015$$
$$\approx 351.$$