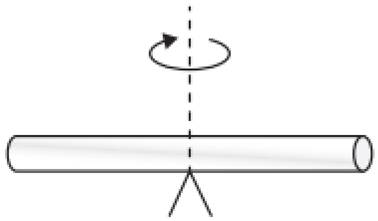
Question 1

A bar rotates horizontally about its centre, reaching a maximum angular velocity in six complete rotations from rest.



The bar has a constant angular acceleration of 0.110rads^{-2}. The moment of inertia of the bar about the axis of rotation is 0.0216kgm^2.

(a) Show that the final angular velocity of the bar is about 3rad^{-1} [2]

(b) Draw the variation with time t of the angular displacement θ of the bar during the acceleration [1]

(c) Calculate the torque acting on the bar while it is accelerating. [1]

(d) The torque is removed. The bar comes to rest in 30 complete rotations with constant angular deceleration. Determine the time taken for the bar to come to rest [2]

a) $\omega_f^2 - \omega_i^2 = 2\cdot a\cdot\theta$

$\omega_f^2 - 0 = 2\times 0.11 \times 12\pi$

$\omega_f = 2.879 = 3\text{ rad s}^{-1}$

b)

c) $\tau = I\alpha$

$= 0.0216 \times 0.110$

$= 2.376\times10^{-3}$

d) $\omega_f^2 - \omega_i^2 = 2\alpha S$

$0 - 2.88^2 = -2\cdot\alpha\cdot 60\pi$

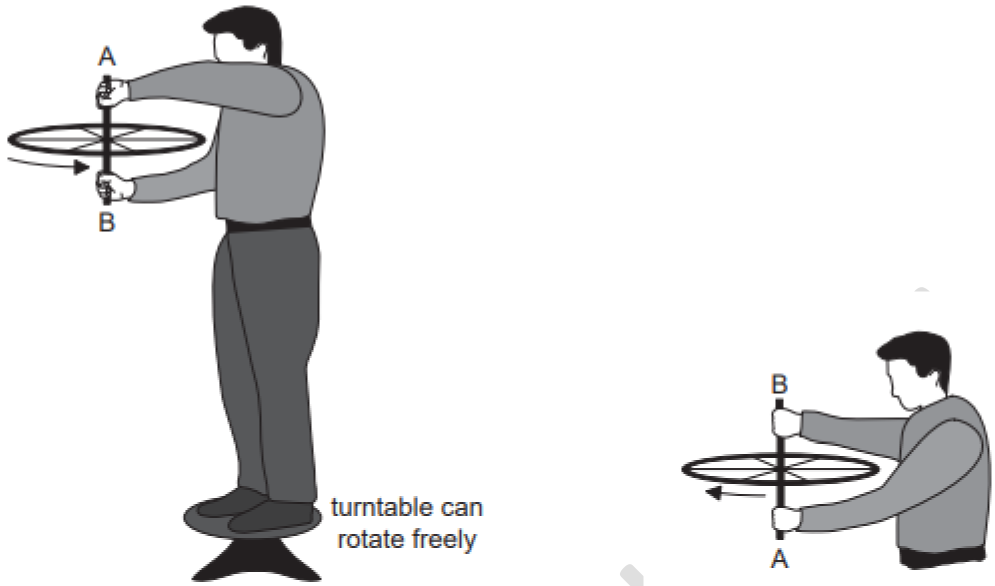
$\Rightarrow \alpha = \frac{2.88^2}{2\times 60\pi} = 0.022\text{ rad s}^{-2}$

$t = \frac{\omega_f - \omega_i}{\alpha}$

$= \frac{0 - 2.879}{-0.022} = 131$ Seconds

Question 2

The first diagram shows a person standing on a turntable which can rotate freely. The person is stationary and holding a bicycle wheel. The wheel rotates anticlockwise when seen from above.



The wheel is flipped, as shown in the second diagram, so that it rotates clockwise when seen from above

(a) Explain the direction in which the person-turntable system starts to rotate. [3]

(b) Explain the changes to the rotational kinetic energy in the person-turntable system. [2]

a) person rotates anticlockwise as it gain angular momentum in opposite direction in accordance with law of conservation of angular momentum

b) Rotational KE is increased as the person-turntable system also now rotate in anticlockwise direction. as energy is provided by the person in changing direction.

Question 3

A solid sphere of radius r and mass m is released from rest and rolls down a slope, without slipping. The vertical height of the slope is h. The moment of inertia I of this sphere about an axis through its centre is 2/5 mr^2

Show that the linear velocity v of the sphere as it leaves the slope is $\sqrt{\frac{10gh}{7}}$ [3]

Total energy at top = Total energy at bottom.

$\Rightarrow (mgh)_{top} = (K_R + K_L)_{Bottom}$.

$mgh = \frac{1}{2} I \omega^2 + \frac{1}{2} m v^2$

$\cancel{m} g h = \frac{1}{2} \times \frac{2}{5} \cancel{m} \cancel{r^2} \times \frac{v^2}{\cancel{r^2}} + \frac{1}{2} \cancel{m} v^2$

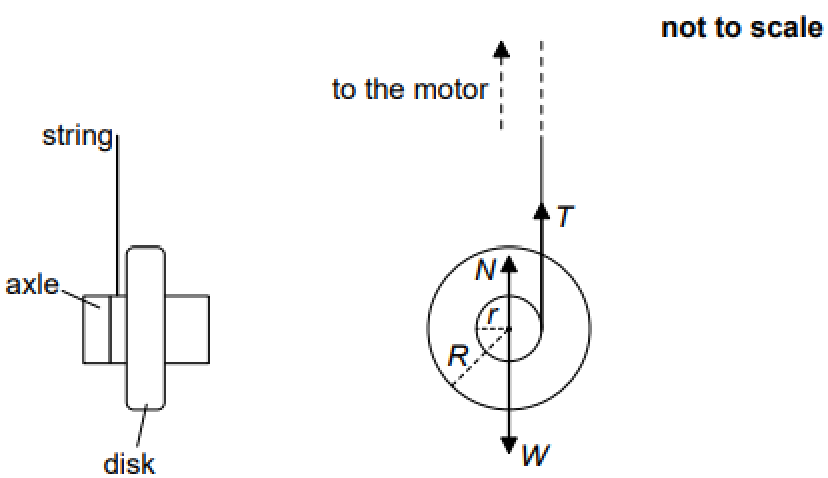
$\Rightarrow gh = \frac{2v^2}{10} + \frac{1}{2} v^2$

$= \frac{2+5}{10} v^2$

$\Rightarrow v = \sqrt{\frac{10gh}{7}}$

Question 4

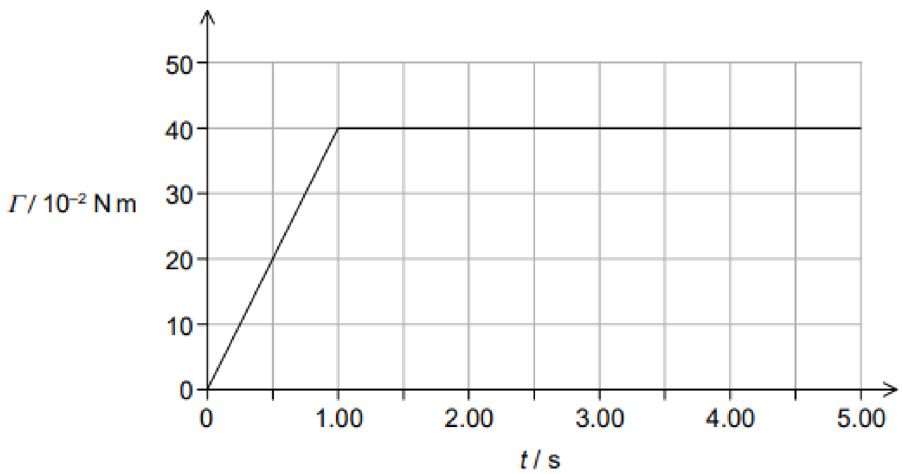
A flywheel is made of a solid disk with a mass M of 5.00kg mounted on a small radial axle. The mass of the axle is negligible. The radius R of the disk is 6.00cm and the radius r of the axle is 1.20cm. A string of negligible thickness is wound around the axle. The string is pulled by an electric motor that exerts a vertical tension force T on the flywheel. The diagram shows the forces acting on the flywheel. W is the weight and N is the normal reaction force from the support of the flywheel.



The moment of inertia of the flywheel about the axis is $I = 0.5\ MR^2$.

(a) State the torque provided by the force W about the axis of the flywheel. [1]

(b) The flywheel is initially at rest. At time t = 0 the motor is switched on and a time-varying tension force acts on the flywheel. The torque Γ exerted on the flywheel by the tension force in the string varies with t as shown on the graph



(i) Identify the physical quantity represented by the area under the graph. [1]

(ii) Show that the angular velocity of the flywheel at t = 5.00s is 200rads^{-1}. [2]

(iii) Calculate the maximum tension in the string. [1]

(c) At t = 5.00s the string becomes fully unwound and it disconnects from the flywheel. The flywheel remains spinning around the axle.

(i) The flywheel is in translational equilibrium. Distinguish between translational equilibrium and rotational equilibrium. [2]

(ii) At t = 5.00s the flywheel is spinning with angular velocity 200rads^{-1}. The support bearings exert a constant frictional torque on the axle. The flywheel comes to rest after 8.00 × 10^3 revolutions. Calculate the magnitude of the frictional torque exerted on the flywheel. [3]

a) zero because the weight W is along the line of axis of rotation.

b) $\tau = I.\alpha$

$\tau = I.\frac{\omega}{t}$ | $\Rightarrow \tau \times t = I\omega = L$ (angular momentum) (b)(ii) $A = A_1 + A_2 = I\omega$

$= \frac{1}{2} \times b \times h + L \times B = I\omega$

$= \frac{1}{2} \times 1 \times 40 \times 10^{-2} + 40 \times 10^{-2} \times 4 = 0.5 m r^2 \omega$

$\Rightarrow \omega = \frac{0.2 + 1.6}{0.5 \times 0.06^2 \times 5} = 200 \text{ rad s}^{-1}$

c(i) Translational equilibrium is when sum of all forces is zero

Rotational equilibrium is when sum of all torque is zero.

c(ii) $\alpha = \frac{\omega_f^2 - \omega_i^2}{2\theta} = \frac{0 - 200^2}{2 \times 8 \times 10^3 \times 2\pi} = 0.398 \text{ rad s}^{-2}$

$\tau = I\alpha$ $\quad$ $\} \; I = 0.5 m r^2$

$= 0.5 \times 5 \times 0.06^2 \times 0.398$

$= 3.58 \times 10^{-3}$ Nm